American University of Beirut
Math 204
Quiz II – (Fall 2016)

Time 60 minutes.

Name: ____________________ ID#: ____________________

Circle your problem solving section number below:

- **Instructor:** Ms Joumana Tannous

Section 1 @ 1:00 M   Section 2 @ 11:00 M   Section 3 @ 4:00 M

- **Instructor:** Mrs Maha Itani-Hatab

Section 4 @ 11:00 Tu   Section 5 @ 8:00 Tu   Section 6 @ 12:30 Tu

- **Instructor:** Ms. Michella Bou Eid

Section 7 @ 12:30 Th   Section 8 @ 2:00 Th   Section 9 @ 5:00 Th

- **Instructor:** Ms Najwa Fuleihan

Section 10 @ 8:00 Tu   Section 11 @ 2:00 Tu   Section 12 @ 11:00 Tu

Solution

Page	Grade
2	/ 23
3	/ 25
4	/ 28
5	/ 24
Total	/ 100

1) To buy a computer system, a customer can choose one of 4 monitors, one of 2 keyboards, one of 7 computers and one of 3 printers. How many possible computer systems a customer can choose?

(3 pts)

$4 \times 2 \times 7 \times 3$

2) A committee of seven consisting of a chairman, a vice chairman, a secretary, and four other members is to be chosen from a class of 20 students. In how many ways can this committee be chosen?

(4 pts)

${}_{20}P_3 \times {}_{17}C_4$

3) Three-digit numbers are formed using the digits 2, 3, 4, 5, and 7. How many such numbers can be formed if the numbers are:

i. greater than 700 and repetition is allowed?

(8 pts)

$1 \times 5 \times 5$

ii. even and repetition is not allowed?

$4 \times 3 \times 2$

iii. less than 700, divisible by 5 and repetition is not allowed?

$3 \times 3 \times 1$

4) The phone number of Jad is 03 386 350. How many different passwords can he form using the eight digits of his phone number?

(3 pts)

$$\frac{8!}{2!\,3!\,1!\,1!\,1!}$$

5) A company has 2844 employees. Is it possible to give each employee an ID number that consists of one letter followed by two digits? Explain.

(5 pts)

The number of ways to form the ID is

$26 \times 10 \times 10 = 2600$

Since $2600 < 2844$ that is number of ID's < number of employees

It is not possible to give each employee an ID number.

6) Solve for n $\quad 2\,{}_{n}P_{3} = {}_{2n}C_{3}$ $\quad n \geq 3 \quad 2n \geq 3$

(7 pts)

$$\frac{2\,n!}{(n-3)!} = \frac{(2n)!}{(2n-3)!\,3!} \to 2\,n(n-1)(n-2)(n-3)$$

$$2\,\frac{n(n-1)(n-2)\cancel{(n-3)\cdots 2\times 1}}{\cancel{(n-3)(n-4)\cdots 2\times 1}} = \frac{(2n)(2n-1)(2n-2)\cancel{(2n-3)\cdots \times 3\times 2\times 1}}{\cancel{(2n-3)(2n-4)\cdots 2\times 1}\cdot 3\times 2\times 1}$$

$$6\times \cancel{2n}(n-1)(n-2) = \cancel{2n}(2n-1)(2n-2) \quad \text{because } n \neq 0 \ (n \geq 3)$$

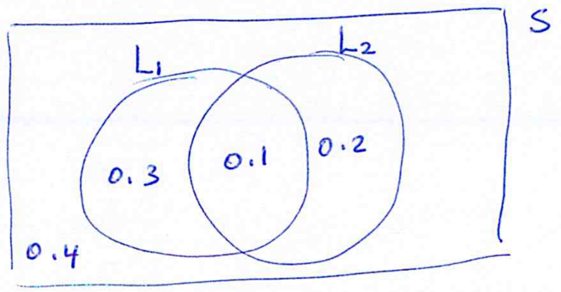
$$6(n^2-3n+2) = 4n^2-4n-2n+2$$

$$2n^2-12n+10=0 \to n^2-6n+5=0 \to (n-1)(n-5)=0 \quad \begin{cases} n=1 \text{ rejected} \\ n=5 \text{ accepted} \end{cases}$$

7) Suppose that your street has two traffic lights. The chance that the first light is red is 0.4, the chance that the second light is red is 0.3 and the chance that both are red at the same time is 0.1.

(18 pts)

a. Draw a Venn diagram to represent the given.



b. What is the probability that:

i. the first or the second light are red?

$$P(L_1 \cup L_2) = P(L_1) + P(L_2) - P(L_1 \cap L_2)$$
$$=$$

ii. the first light is red and the second light is not?

$$P(L_1 \cap L_2') = 0.3$$

iii. exactly one of the lights is red?

$$P(L_1 \cap L_2') + P(L_2 \cap L_1') = 0.3 + 0.2 = 0.5$$

iv. the second light is red given that the first light is red?

$$P(L_2 | L_1) = \frac{P(L_1 \cap L_2)}{P(L_1)} = \frac{0.1}{0.4} = \frac{1}{4}$$

c. Are the events "the first light is not red" and "the second light is not red" mutually exclusive?

$$P(L_1' \cap L_2') = P(L_1 \cup L_2)' = 1 - P(L_1 \cup L_2) = 0.4 \neq 0$$

$\therefore$ they are not mutually exclusive

d. Are the events "the first light is red" and "the second light is red" independent?

$$P(L_1 \cap L_2) = 0.1$$

$$P(L_1) \times P(L_2) = 0.4 \times 0.3 \neq 0.1 = P(L_1 \cap L_2)$$

They are not independent

8) A bag contains 2 red, 3 green and 4 blue balls. Two balls are drawn at random from the urn, what is the probability that none of the balls drawn is blue?

(3 pts)

$$\frac{5}{9} \times \frac{4}{8} \quad \text{or} \quad \frac{{}_4C_0 \times {}_5C_2}{{}_9C_2}$$

9) The probabilities that three different students A, B and C will get a scholarship are respectively 0.6, 0.7 and 0.8. Assuming independence.

(5 pts)

i. Find the probability that only one will get a scholarship.

$$0.6 \times 0.3 \times 0.2 + 0.4 \times 0.7 \times 0.2 + 0.4 \times 0.3 \times 0.8 = 0.188$$
$$0.036 + 0.056 + 0.096$$

ii. If only one got a scholarship, what is the probability that he is student A?

$$P(A \mid \text{only one}) = \frac{0.6 \times 0.3 \times 0.2}{0.188} = 0.19$$

10) In a restaurant seventy percent of people order only Chinese food and thirty percent order only Italian food. A group of three persons enter the restaurant, what is the probability that at least two of them order Italian food.

(3 pts)

$$P(X \geq 2) = {}_3C_2 (0.3)^2 (0.7)^1 + {}_3C_3 (0.3)^3 (0.7)^0$$

11) The probability that a man will purchase sports cars is 0.6. If 10 sports car owners are randomly selected, find the probability that:

(17 pts)

i. exactly 7 are men.

$$P(X=7) = {}_{10}C_7 (0.6)^7 (0.4)^3$$

ii. exactly 5 are women

$$P(Y=5) = {}_{10}C_5 (0.6)^5 (0.4)^5$$

iii. at least 2 are men

$$P(X \geq 2) = 1 - P(X=0,1) = 1 - \left[{}_{10}C_0 (0.6)^0 (0.4)^{10} + {}_{10}C_1 (0.6)^1 (0.4)^9\right]$$

iv. between 4 and 6 are men

$$P(4 < X < 6) = P(X=5) = {}_{10}C_5 (0.6)^5 (0.4)^5$$

v. more than 7 men given that at most 9 are men

$$P(X > 7 \mid X \leq 9) = \frac{P(7 < X \leq 9)}{P(X \leq 9)} = \frac{P(X=8,9)}{1 - P(X=10)} = {}_{10}C_8 (.6)^8 (.4)^2 + {}_{10}C_9 (.6)^9 (.$$

vi. no less than 8 are men

$$P(X \geq 8) = P(X=8,9,10) = {}_{10}C_8 (.6)^8 (.4)^2 + {}_{10}C_9 (.6)^9 (.4)^1 + {}_{10}C_{10} (.6)^{10} (.$$

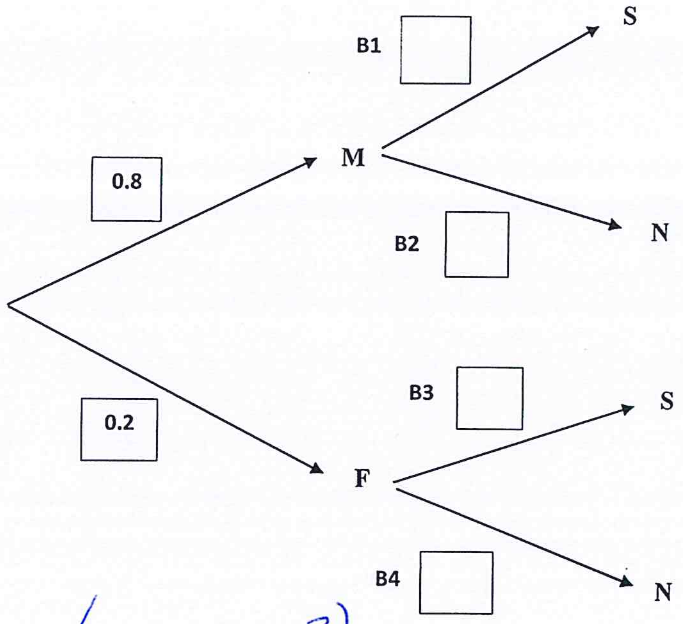
- In a sample of 120 sports cars owners, what is the expected number of men?

$$\mu = 120 \times 0.6 = 72$$

(24 pts)

12) The question " Do you smoke? " was asked to a sample of 100 people. The results are shown in the following table. One person is selected at random.

	Male	Female	Total
Smoker	16	3	19
Nonsmoker	64	17	81
Total	80	20	100



a. Use the table to fill in the boxes of the tree diagram by the corresponding probabilities. Justify your answers.

Box B1: $P(S/M) = \frac{P(S\cap M)}{P(M)} = \frac{16}{80} = 0.2$

Box B2:

$P(N/M) = \frac{P(N\cap M)}{P(M)} = \frac{64}{80} = 0.8$ (or $1-0.2=0.8$)

Box B3:

$P(S/F) = \frac{P(S\cap F)}{P(F)} = \frac{3}{20} = 0.15$

Box B4:

$P(N/F) = \frac{P(N\cap F)}{P(F)} = \frac{17}{20} = 0.85$ (or $1-0.15=0.85$)

b. Find the probability that the selected person is:

i. a female and nonsmoker

$P(F\cap N) = \frac{17}{100} = 0.17$

ii. a male or smoker

$P(M\cup S) = P(M) + P(S) - P(M\cap S) = \frac{80}{100} + \frac{19}{100} - \frac{16}{100} = \frac{83}{100} = 0.83$

iii. neither a male nor a smoker

$P(M'\cap S') = P(M\cup S)' = 1 - P(M\cup S) = 1 - \frac{83}{100} = \frac{17}{100} = 0.17$

c. If the selected person is nonsmoker, what is the probability that he is a male?

$P(M/N) = \frac{P(M\cap N)}{P(N)} = \frac{64}{81} = 0.7901$

d. Are the events "is nonsmoker" and "is a female" collectively exhaustive?

$P(N\cup F) = P(N) + P(F) - P(N\cap F) = \frac{81}{100} + \frac{20}{100} - \frac{17}{100} = \frac{84}{100} = 0.84 \neq 1$

$P(N\cup F) \neq 1 \longrightarrow N\cup F \neq$ Sample Space

not collectively exhaustive